HANS VAN DITMARSCH

Serious Play

All my life I have been playing games, and all my professional life I have worried whether playing games is a serious enough occupation. When I was working on a doctorate on the board game Cluedo, I had trouble convincing my friends that I was actually getting paid to do this research. In this article, I make the point that there are important insights to be gained by applying various scientific disciplines to the analysis of frivolous pastimes. I focus therefore on the serious nature of games and show, through my personal history with games – or rather my attempts to change the rules of those games – how probability theory, game theory, and epistemic logic infuse these playful activities. We'll look one by one at Risk, Cluedo, Pit and Sudoku, at their setting and rules as well as at 'serious' insights which probably will not have struck their casual players.

1. PLAYING GAMES

1.1 Risk

Risk is a war game wherein a board represents the world, and continents have to be conquered. The different countries and regions on the world's continents can be occupied by armies. Opponents in neighbouring countries can attack one another by designating a number of attacking armies. Winning means that all opposing armies in the neighbouring country are killed in battle. Battles are determined by throwing dice. The opponents both throw the dice, and the higher pair wins. The attacker may use up to three armies – throw up to three dice – whereas the defender may defend with up to two armies – two dice. The highest of the attacker's throw is matched with the highest of the opponent's, and the opponent of whomever's is higher, is removed from the board. Then the next highest throw is matched with the next highest of the opponent, and again the higher throw wins.

For example, you attack Brazil from Argentina with three armies, and your opponent has two armies in Brazil. If you throw 5, 3, and 2, and your opponent throws 2 and 1, you kill both defending armies: 5 wins from 2 and 3 wins from 1. Your Argentinean armies are now moved over to Brazil. If the opponent had thrown 4 and 2, you still kill both defending armies: 5 wins from 4 (so 3 cannot be beaten by 4), and 3 wins from 2. If your opponent throws 4 and 3, you each lose an army: 3 against 3 means that the defender wins. You will have to throw again.

Is it better for you, the attacker, if your highest dice are 4 and 6, or if they are 5 and 5? If the defender throws two dice there are 36 different outcomes. In the case of attacking (6,4) the defender defeats both his opponent's armies if he throws (6,4), (6,5), or (6,6); and both lose an army if the defender throws (5,5), (5,4), and (4,4). In the remaining 30 cases, the defender loses both armies. In the case of attacking (5,5), the defender wins two if he throws (6,6), (6,5), or (5,5); and both players lose one if the defender throws (5,4), (5,3), (5,2), or (5,1). In the remaining 29 cases, the defender loses both armies. Our initial analysis shows that the odds with (6,4) are slightly more in favour of the attacker than with (5,5). But this also depends on the rules with which you play Risk.

According to the rules with which I am most familiar, the defender may choose to defend a single army or two armies (that is to say, throw one die or two) after the attacker has thrown his dice. For example, if the attacker's highest are (5,5), and the defender throws just one die, he wins the attacker's army if he throws 5 or 6, and loses in the four other cases: the probability of losing is therefore 4/6 (66%), much less than the 29/36 (81%) that the defender confronted before. Also, the defender now loses *one* army only, and still has one left.

On the other hand, defending with a single die in the case of an attacking (6,4) is a decidedly less attractive alternative than in the case of (5,5): the probability of defeating a 6 is 1/6 only, namely if the defender also throws a 6. This demonstrates that the odds with (6,4) are even more in favour of the attacker compared to (5,5), than we already observed.

By means of probabilities one can show that defending with one die instead of two dice is always better when the attacker has (5,5), but for (6,4) this depends on the number of his remaining armies. If the defender has only two armies left, defending with two dice is more attractive than defending with one. This is because after losing one army in the next round he no longer has a choice to defend with one or two. For more information on the odds in Risk, see the "Science News Online" item,[1] or JA Osborne's mathematically thorough article *Markov chains for the Risk board game revisited*.[2] The choice between defending with one die or two dice is discussed on Owen Lyne's FAQ for the board game Risk.[3]

Although the probabilistic analysis is useful, the game is so complex that statistical knowledge alone will not make one a better player: from the higher perspective of realising the world's conquest (or at least three of its continents, one of the possible winning conditions in Risk); and it is always very unclear if the cost in armies of winning new territory is worth that benefit. Of far more importance is one's ability to make alliances with other players, bluff your way through a threatening board position by strategically posting armies on border countries of continents, etc.

1.2 Cluedo

Imagine a country mansion with a couple of partying guests. Suddenly the host is discovered lying in the basement, murdered. The guests decide to find out among themselves who committed the murder. The body is discovered by the butler, under suspicious circumstances that indicate that the location is not the actual murder room. In order to solve the murder, it is required to find out who the murderer is, what the murder weapon was, and in which room the murder was committed. The butler is exonerated, and the six guests are therefore the suspects.

Figure 1: Green has done it with a candlestick in the ballroom: the cards.

The 'murder detection game' Cluedo is played on a game board with a picture of the house, with the nine rooms in it and 'paths' leading in a certain number of steps from one room to another. Typically, there are six players. There are six guest cards, six weapon cards, and nine room cards. The three categories of cards are shuffled separately. One suspect card, one weapon card and one room card are blindly drawn and put apart. These 'murder cards' represent the actual murderer, the murder weapon and the murder room. *The first player to guess those cards wins the game*. All remaining cards are shuffled together. They are then dealt to the players. Every player gets three cards.

Some player starts the game. A player's move consists of throwing the dice and (trying to) reach a room by walking a pawn over the game board, and then, if a room has been reached, *voicing a suspicion* about it, i.e. about a guest, a weapon and that particular room. This is in fact a request for the confession of ownership of one of those three cards. The other players now respond to the request in clockwise fashion: either a player doesn't have any of the requested cards, and says so, after which the next player responds to the request; or a player holds at least one of the requested cards, he shows exactly one of those to the requesting player only, and no further responses may be gathered. You may now either *end your move* (who is next in turn is again determined clockwise) or, if you think you know enough to guess the murder cards, make an accusation.

An accusation is also the combination of a suspect, a weapon and a room card, but it plays a different role in the game than a suspicion does. Each player can make an accusation only once. It is not voiced but written down. The accusing player then checks the three murder cards, without showing them to others. If the accusation is false, that player has lost and the game continues. If the cards match the accusation, it is successful. The first player who makes a successful accusation wins the game.

The precise information change due to a card show action to the requesting player only is fairly complex to describe, for details see my earlier piece on the description of game actions.[4] For now, let us focus on the game action of 'ending your move'. For fully rational players, ending your move corresponds to announcing that you are ignorant of the murder cards. This may be informative (in the sense that certain combinations of players and cards are eliminated) for the remaining players. For example, consider the following different scenarios:

Player 1, who starts the game, reaches the kitchen in his first move. He voices the suspicion 'I think that Scarlett has committed the murder in the kitchen with a knife'. None of the other players can show a card. Player 1 (confidently) writes down an accusation, checks the murder cards and announces that he has won. (1)

Player 1, who starts the game, reaches the kitchen in his first move. He voices the suspicion 'I think that Scarlett has committed the murder in the kitchen with a knife'. None of the other players can show a card. Player 1 now ends his move. (2)

In scenario (1) the murder cards are kitchen, Scarlett, and knife; and player 1, by mere incredible luck, accidentally asked for precisely these cards. When nobody showed a card, he therefore *knew* that the requested cards must be the murder cards, and wins. But in scenario (2), the other players can deduce that player 1 holds at least one of the requested cards from the act of 1 ending his move: because if he *had not* held one of the requested cards, he would have known what the murder cards were and would have made the accusation, as in (1). But he didn't. So he *must* have one of those cards.

Scenarios (1) and (2) seem very unusual. But in fact this 'end your move' action always happens when a player passes his turn to the next player, and may then have similar informative results – and this insight is novel. There is one hitch: real players may not be aware of all the deductive consequences of their knowledge. So they might end their turn, even though they *could* have known the murder cards and won, by making an accusation. The other players, reasoning from the assumption that that player could not have known, then reason from incorrect assumptions which may lead to false conclusions and even false accusations: so they will lose after all. Now it is not 'legal' in Cluedo not to show a card if you have one, but it is perfectly legal not to be a perfect logician and not make an accusation even if you *should* have made one. So the insight, while novel, assumes players who know exactly what they are doing and is therefore dangerous to apply to fallible human players.

1.3 Pit

The object of Pit is to corner the market on Barley, Corn, Flax, Hay, Oats, Rye and Wheat by trading cards with other players. Pit can be played by three to seven players. There are nine cards in each suit. If three play, use only three complete suits. If four play, use four complete suits, etc. Use the complete Pit deck for seven players. Place the trading bell in the center of the table and select one player to shuffle the deck and deal nine cards to each player. The Dealer should allow the players 30 seconds to sort their cards and decide mentally on which commodity (Wheat, Rye, Oats, etc.) they will attempt to corner. When the cards have been sorted, the Dealer strikes the bell and announces, 'The Exchange is open.' To trade, a player takes from his hand one to four cards of the same suit, holds the cards up so that the suits do not show and calls out, 'Trade One! One! One!' or 'Two! Two! Two!' or 'Three! Three! Three!' or 'Four! Four! Four!' depending on the number of cards being traded. Players may trade an equal number of cards of the same suit with another player calling for the same number of cards. Trading continues until one player gets nine cards of the same suit. That player must quickly ring the bell and call out, 'Corner on Wheat!' (or whatever the commodity may be).

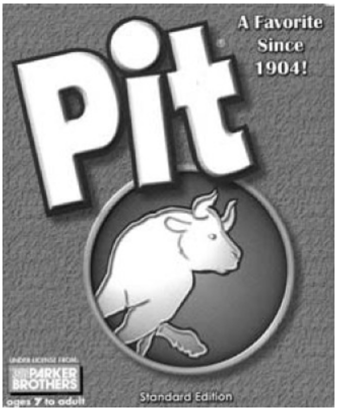

Figure 2: The Pit game.

Consider a simplification of the Pit game for only three players Anne, Bill, and Cath (*a*, *b*, and *c*) who each hold two cards from a pack consisting of two Wheat, two Flax, and two Rye cards. A commodity corresponds to a suit. The suits are abbreviated as *w*, *x*, *y*. For the card deal where Anne holds a Wheat and a Flax card, Bill a Wheat and a Rye card, and Cath a Flax and a Rye card, we write *wx.wy.xy*, etc. As the cards in one's hand are unordered, *wx.wy.xy* is the same deal of cards as *xw.wy.xy* – but for improved readability we will always list cards in a hand in alphabetical order.

There are 21 different card deals, but there are only six deals that do not involve a corner (e.g., if the deal is *ww.xy.xy*, Anne declares a corner in Wheat and wins). Consider the card deal *wx.wy.xy*. In this game state players seem to be 'equally well' informed: all players only know that the other two players also have two cards of different suits. All players now offer one card for trade with another player. For simplicity we assume that they want to trade with all other players. Suppose Anne and Bill are selected to trade (in other words, we imagine all three players shouting 'One! One! One!' and then Ann and Bill actually trading). The four possible outcomes of that trade (for given deal *wx.wy.xy*) are *wx.wy.xy*, *xy.ww.xy*, *ww.xy.xy*, and *wy.wx.xy*. For example, if Anne had selected Flax when making the offer, and Bill Wheat, the resulting card deal is *ww.xy.xy*. In that case Anne declares a corner in Wheat and wins. If the resulting deal is *xy.ww.xy*, Bill declares a corner in Wheat and wins. If the result is *wx.wy.xy* or *wy.wx.xy*, then it becomes clear to Cath that neither Anne nor Bill have achieved a corner, because they failed to declare it, and a further move has to be made in the game, possibly ad infinitum.

We now get to the point of this story. When, after the trade, the card deal is again *wx.wy.xy*, the game state *appears* to be the same as it was originally, when the cards had just been dealt. But in fact this is *not* the case: Anne and Bill, but not Cath, now happen to know what the card deal is, and they can use that information when planning their next action. Let us first explain why Anne and Bill are so knowledgable.

Anne now knows that Bill has a Wheat card, because she just gave him her own. But Anne can also deduce Bill's other card: it cannot also be Wheat, because she received Wheat, and if Bill's other card *had* been Wheat, he would already have declared a corner in Wheat and would not have offered a card for trade. But it cannot be Flax either, because then Cath would have had two Rye cards initially and would already have declared a corner in Rye. She didn't. So Bill's other card must be Rye. But then Cath holds the two remaining cards: Flax and Rye. In other words: Anne knows that the deal of cards is *wx.wy.xy*. Bill can reason in the same way as Anne, and therefore also knows the deal of cards.

Anne can therefore distinguish between 'the card in her hand that she has in common with Bill, with whom she just traded', namely her Wheat card, and 'the card in her hand that she has in common with Cath, with whom she did not just trade', namely her Flax card. Something similar holds for Bill. Therefore, both can make a strategic choice for their next offer. Cath has not gained factual knowledge (i.e., knowledge about card ownership) from Anne and Bill trading, but she now knows that Anne and Bill know the deal of cards. So she still has learnt something that, in principle, may determine her next offer.

To conclude, game states with the same distribution of cards may differ in their epistemic properties: players may know more or less about one another in different game states. This information can then be used in order to win. How? Even though we could continue, here we stop our explanations again, as we have already reached our goal. The study of determining the best next move in view of all other players also doing that, is called 'game theory', an area of economics (see Section 3). A game theoretical investigation on Pit is found in my 2004 piece on the game theory of Pit.[5] The complex and often simultaneous actions in 'real life' Pit are investigated in M Purvis and his colleagues' work, a study in requirements engineering for electronic market simulations.[6]

1.4 Sudoku

Sudoku is the well-known game consisting of a 9 by 9 matrix, subdivided in nine 3 by 3 matrices (called 'blocks'), where each row, column, and block should contain the numbers 1 to 9. There are very many different ways to fill a 9 by 9 square in this manner; in a Sudoku 'puzzle' many entries are already given ('givens') such that only *one* way is possible to add remaining numbers and fill the square. Sudoku is known to have conquered the world – or rather the world's newspapers: *The Otago Daily Times* also features a daily Sudoku – starting from Japan. Less known may be why such a game could have become popular in Japan instead of an arbitrary other country. A friend who lived in Japan in the early 90s, when Sudoku was already popular there but not in the rest of the world, explained to me why this is not accidental but obvious. Crossword puzzles are also quite popular as daily newspaper entertainment in many countries. Now try to imagine a Japanese newspaper featuring a crossword puzzle for its readers. How many characters are there to choose from, when filling a square? Instead of 26 letters in the Roman alphabet, we are now talking about over 10,000 Chinese (and Japanese) characters. Not very practical, nor entertaining. But as the Japanese have adopted the Arabic decimal system, in Western notation, in other words the digits 0, 1,...9, number puzzles were the obvious more playable alternative. Therefore, *only* in Japan (or some other East-Asian country using the Chinese alphabet) was Sudoku free of competition from crosswords, allowing it to become popular.

Recall that each Sudoku puzzle must have a unique solution. If no numbers are given, there are millions of millions of different solutions. If *all* (9 times 9 equals) 81 numbers are given, there is a unique solution, but that is silly because in that case there are no cells without numbers so it is no longer a puzzle. How many numbers must be given? This, of course, depends on their position. Even when all but four of the numbers are given, there may be more than one solution. For example, suppose two 4s and two 6s are missing, and the empty cells are in the positions given below. It will be clear that if

4	6							
6	4							

provides a solution, then so does

6	4							
4	6							

because in both cases rows 3 and 5 contain a 4 and a 6, as well as columns 1 and 2, and the corresponding 3*3 blocks as well. The maximum number of givens for which there may be more than one solution is therefore (81 minus 4 equals) 77. The minimum number of givens for which there is a configuration with a unique solution is unknown. It is at most

17. Sudoku puzzles have been constructed with only 17 given numbers but with a unique solution. However, it has not been proved that this is impossible with a smaller number of givens. For more information, see the very informative Wikipedia item http://en.wikipedia.org/wiki/Sudoku , and for Sudoku puzzles with a minimum number of givens, see http://www.csse.uwa.edu.au/~gordon/sudokumin.php

We now provide the reader with a new constraint to help solving a Sudoku puzzle, the 'unique solution'-constraint, to my knowledge not found in the regular Sudoku manuals – nor on, for example, the above website. Suppose that we are in the process of filling a Sudoku square with new entries and have derived a configuration involving

4	6							
6	*							

where the 4 and the two 6s were not initially given, and * is still blank. We now can deduce that the * cell cannot be the number 4, because if it *were*, there would be *more than one solution*, which is ruled out by the definition of the Sudoku puzzle. One can easily imagine configurations where this is helpful towards solving the Sudoku, and where that 4 would otherwise have been impossible to rule out in the * position.

The more general form of the constraint is as follows:

Consider four non-givens that are the corners of a rectangle in the Sudoku matrix, such that two are found in the same block. If two (diagonally) opposing numbers are the same, then the other two (therefore also opposing) numbers must be different.

This constraint is rather dangerous to apply unless one feels entitled to assume that the Sudoku puzzle designer really knows her business: if the designer mistakenly supplied a puzzle with more than one solution, then applying the 'unique solution'-constraint may result in one not finding any of those. Not so long ago, I solved a Sudoku in the otherwise respectable French newspaper *Le Monde* that had two solutions… The 'unique-solution'-constraint can be said to be based on a higher-order analysis of the game, where the combined rules and restrictions for solving it unintentionally provide yet another restriction.

2. GAME THEORY AND EPISTEMIC LOGIC

In Risk we explained some of the probabilities relevant when throwing dice in order to conquer armies; in other words, we applied *probability theory*. The analysis of Cluedo used what players know about each other and about each other's actions: this can be formalised in an area of logic called *epistemic logic*, also known as the logic of knowledge. In Pit we also used epistemic logical insights, and expanded a bit on *game theory*, a sub-discipline of economics. The next three sub-sections give some information on those academic disciplines, and how they have contributed to solving problems other than those of fun and games.

2.1 Probability Theory

Probabilities have been applied to the analysis of games since times immemorial and in particular since the sixteenth century. Key figures are the sixteenth-century Italian mathematician Girolamo Cardano and the seventeenth-century French philosopher and mathematician Blaise Pascal. Pascal laid the foundation for the theory of probability in correspondence with Pierre de Fermat, another well-known mathematician from that era. This correspondence consisted of five letters and occurred in the summer of 1654. Both Cardano and Fermat investigated what is known as the dice problem.[7] The dice problem asks how many times one must throw a pair of dice before expecting a double six. Of course the answer to this question is not a number! The probability of double six after one throw (with a pair of two dice) is 1/36 (about 3%); the probability of double six after two throws is (1 - (the probability of not having double 6 after two throws)) = $(1 - (35/36)^2) = (1 - 1225/1296) = 71/1296$ which is, roughly, 5.5%. This is of course higher than 1/36, as there is still another chance to throw. Also note that this probability is a lot more than $(1/36)^2$, which amounts to less than 0.1%, this is namely the probability of throwing double six *twice*. And so on, for throwing thrice, four times…Double sixes were, as we saw, already an issue in the Risk game, as a double six beats the opponent's armies. But of course there are many more basic sorts of gambling with just dice where this also plays a role. When playing for money, such knowledge of probabilities may make the difference between winning or losing, an obvious practical concern for regular customers of casinos. In fact, Cardano did play dice for money and even earned a living that way: he thought that gambling for fun was a waste of time.

2.2 Game Theory

Game theory came to the fore as a sub-discipline of economics in the mid-twentieth century. Even though games as such appear rather innocent, there are many other human activities that can be analysed as games, that is to say, as proceduralised interaction between often fairly small groups of individuals, where game moves correspond to choice between well-described alternative actions. For example, opposing generals on a battlefield have to choose between attack and defence; traders playing the stock market have to choose between buying and selling; voters in the election have to choose between different parties; and so on. In all these cases, one models the strategic choices of different players that can act either simultaneously or sequentially, where each individual player's choice depends on the choices of all other players. Game theory came into its own with the classic *Theory of Games and Economic*

Behaviour by von Neumann and Morgenstern,[8] and later by contributions in the 1950s and 1960s by Nobel Prize winner John Nash – also known from his biography, and the resulting film, *A Beautiful Mind*.'[9] Robert Aumann is a more recent Nobel Prize winner in that area. In Aumann's work, game theory and the logic of knowledge (as used for the analysis of the Cluedo game) overlap.[10] For a gentle and very accessible introduction to game theory, see *Fun and Games* by Ken Binmore.[11]

Game theory analyses the general case where a finite number of players can each choose from a set of actions, and typically they are assumed to be able to act simultaneously. This analysis can be complex because, as demonstrated, each player's best choice depends on the choice of the other players. A well-known example is the 'prisoners' dilemma.' Two partners in crime face a tribunal and will both get two years in prison if they deny committing the crime, and both get five years if they confess, but if one denies and the other confesses (to the crime they committed together, so that the confessor can be considered the 'defector'), the denying prisoner gets six years but the confessing prisoner gets away scot-free: 0 years. The 'strategic choice problem' is as follows: Together the prisoners would be better off by collaborating and denying the crime. But for an individual prisoner confessing is the better option if you know that the other will deny, because the confessor then goes free. Unfortunately, this is an option for both prisoners, so they may end up both confessing and both serving 5 years. This is a sub-optimal situation, because they are now both worse off than if they had both denied the crime.

Another example where the behaviour of players can be analysed with the formal tools of game theory is Pit, as already mentioned. The Pit players simultaneously have to offer a number of cards for trade, and decide whom to trade with. Their choice is obviously influenced by the choices of other players: it is another instance of the *strategic choice problem*. In the toy Pit example that we modelled in section 1.3, each player only had two cards. Now imagine each player having three cards. If Anne has three cards of which two are Wheat and one is Rye, is it better for her to offer one Rye or to offer two Wheat cards for trade? On first consideration it seems better that she stick to her Wheat cards, as she will be one short only of a corner. But whether this is 'wise', or in game theoretical terms, 'rational', very much depends on the counter-offers she can expect. As she will get two cards in return for the two she offers, those might as well be two Rye cards, so that one or two short of a corner does not appear to make much of a difference here. So surely, if the options are between not trading and trading with Bill who offers two cards, she will prefer to trade. Unless, of course, she knows that Bill has one Wheat card...This suggests imprecise information and indecision, but that is only because we do not go into details. In principle, it can be precisely calculated what a most rational move in the game is for a given state of the game, for each player.

But the use of game theory is not restricted to the simple playing of card games. It has been applied in, for example, the design of on-line auctions between individual buyers as on Ebay; the sale of mobile phone licenses to telecom companies for billions of dollars; in scenarios to simulate outbreak of war between major world powers (for example, in the 60s in order to anticipate conflicts between the USA and the USSR); and in many other quite serious pursuits.

2.3 Epistemic logic

Of somewhat later date than game theory is the logical discipline known as *epistemic logic*, or the logic of knowledge. The main proponent of this area is the Finnish philosopher Jaakko Hintikka, and the publication starting the field is his book *Knowledge and Belief: An Introduction to the Logic of the Two Notions*.[12] The basic idea is that you know that a proposition is true if, and only if, that proposition is true in all states of the world that you cannot distinguish from the state you are in. For example, in the Pit game Anne *knows* that she holds a Wheat card, because she can see it, yes, but more formally because, based on what she sees, the real card deal has to be one of *wx.wy.xy* and *wx.xy.wy*, and although she cannot distinguish between them with her limited information, the point is that in both of the possible states she holds a Wheat card. But she *does not know* if Bill holds a Wheat card: she considers it possible that Bill holds Wheat, because the actual deal might be *wx.wy.xy*, but she also considers it possible that Bill holds no Wheat, because the actual deal might be *wx.xy.wy*.

There are funny interactions between knowledge and ignorance in scenarios where more agents are present and reason about each other. Assuming that you, the reader, have not seen my garden, it would be informative if I were to tell you: '*You do not know that I have a kowhai tree in my garden*.' But now you know, right? I just told you so! In other words, I can make true statements that become false because they are announced. Such puzzling phenomena were already central at the inception of the field in Hintikka's work, where a separate chapter is devoted to such speech acts.[13] They are known as 'Moore-Sentences', after the British philosopher GE Moore who first observed them.[14]

That such analyses are no mere trivialities but 'serious business' comes to the fore when we think of designing computer systems that understand or generate (written) language. When we speak (as when we write) we often leave out details that can be taken for granted in a 'normal' context of communication. If Anne says to Bill 'You do not know that *P*...' this is only informative for Bill if Anne actually means to say '*P* is true and I am informing you of *P* and this is helpful because you do not know *P*.' So all this extra information has to be taken into account by Bill, even though it is not strictly being said by Anne. Now imagine replacing Bill by a computer. If one intends to have Anne talk to the computer as if it were human, the computer had better understand Anne as that human. So the computer has to make a similar expansion of the information as above. Otherwise, Anne would have to use many more words for interaction with that computer than with a human, which makes the computer inhuman... 'Do you know what time it is?' 'Yes!' '???? Tell me what time it is, you idiot!' '5:30 PM.' She had only asked for knowledge, not for time itself. Such modelling assumptions also lie behind things like automated travel assistants, but at the present state of the art, for other semantic reasons than epistemic logical reasons.

For more information on epistemic logic, see the accessible *Reasoning about Knowledge*.[15] An online available introduction into the dynamic phenomena involved such as true sentences becoming false when announced is contained in the *Australian Journal of Logic* as referenced overleaf. [16]

With these short overviews of the areas of probability theory, game theory and epistemic logic I close my contribution. I hope to have demonstrated that the analysis of 'real games' involves features clearly linking them to scientific sub-disciplines, thus revealing their 'serious' nature. But let this not keep you away from the real fun of actually playing them, and losing yet again, to your 12-year old child or grandchild.

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1 I Peterson, "Improving the Odds in RISK", *Science News On-line*, 164(2). http://www.sciencenews.org/articles/20030712/mathtrek.asp. last visited October, 2006.

2 JA Osborne, "Markov Chains for the Risk Board Game Revisited", *Math Magazine*, 76(2):129-135, 2003.

3 O Lyne, "Risk FAQ - Version 5.61", Owen Lyne's Home page, http://www.kent.ac.uk/IMS/personal/odl/riskfaq.htm#3.1 last visited October, 2006.

4 HP van Ditmarsch, "The Description of Game Actions in Cluedo", in L A Petrosian and VV Mazalov (eds), *Game Theory and Applications*, Vol. 8: 1-28 (Commack New York: Nova Science Publishers, 2002).

5 HP van Ditmarsch. "Some Game Theory of Pit", in C Zhang, HW Guesgen, and WK Yeap (eds), *Proceedings of PRICAI 2004* (Eighth Pacific Rim International Conference on Artificial Intelligence, GuiLin: Springer, 2004), LNAI 3157: 946-947.

6 M Purvis, M Nowostawski, S Cranefield, and M Oliveira, "Multi-agent Interaction Technology for Peer-to-peer Computing in Electronic Trading Environments", in G Moro, C Sartori, and M Singh (eds), *AP2PC, LNCS 2872* (Springer, 2004), 150-161.

7 M Kline, *Mathematical Thought from Ancient to Modern Times* (London: Oxford University Press, 1972).

8 J von Neumann and O Morgenstern, *Theory of Games and Economic Behavior* (New York: Princeton University Press, 1944).

9 S Nasar, *A Beautiful Mind* (London: Simon & Schuster, 1998).

10 RJ Aumann,"Agreeing to Disagree", *Annals of Statistics*, 1976, 4(6):1236--1239; RJ Aumann and A Brandenburger, "Epistemic Conditions for Nash Equilibrium", *Econometrica*, 1995, 63:1161-1180.

11 K Binmore. *Fun and Games* (Lexington MA: DC Heath,1992).

12 J Hintikka. *Knowledge and Belief: An Introduction to the Knowledge of the Two Notions* (Ithaca, NY: Cornell University Press, 1962).

13 Ibid.

14 GE Moore, "A Reply to My Critics", in PA Schilpp, *The Philosophy of GE Moore*, *The Library of Living Philosophers*, Vol. 4 (Evanston Illinois: Northwestern University, 1942)< 535-677.

15 R Fagin, JY Halpern, Y Moses and MY Vardi, *Reasoning about Knowledge* (Cambridge Mass.: MIT, 1995).

16 HP van Ditmarsch, W van der Hoek and BP Kooi, "Playing Cards with Hintikka: An Introduction to Dynamic Epistemic Logic", *The Australasian Journal of Logic*, 2005, 3: 108-134, see http://www.philosophy.unimelb.edu.au/ajl/2005/

Hans van Ditmarsch is a lecturer in the Computer Science Department of the University of Otago, New Zealand. His current research focuses on the dynamics of knowledge, information-based security protocols, modal logics for belief revision, proof tools for epistemic logic, combinatorics, and computer and information science education.